# A Numerical Study of Radiation Heat Transfer in Planar Solid

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*ABSTRACT :*
*Radiative transport within the electrode and electrolyte layers, as well as surface-to-surface radiation within the fuel and oxygen flow channels, has the potential to greatly influence temperature fields and overall operating conditions of solid oxide fuel cells (SOFC). Radiation from the stack to the environment, including heat losses through insulation, must be accounted for in the plant design, and is very important for effective thermal management of the high temperature stack. On the other hand ,a parametric study was conducted to study the influence of temperature distribution and radiation effect. The problems of radiation and conduction are solved with finite-volumes. Consequently, the authors newly develop a two dimensional simulation code of the planar SOFC stack, and the detailed effect of the radiation heat transfer is investigated .This is because the thermal conductivity of the cell materials made of ceramics is very small, and the central part of the cell stack is almost free form the influence of radiation heat transfer. The combination between non-gray radiation heat transfer and convection-conduction heat transfer is studied.*

*Key-words: Solid Oxide Fuel Cell, radiation modeling, Heat transfer, Temperature distribution.*

## Introduction

Solid oxide fuel cells (SOFC) operate at temperatures of the order of 600–1000 ◦C [1]; thus, radiation heat transfer must be given special consideration in thermal modeling efforts, including stack thermal management and materials development. During the last decades, a number of increasingly detailed theoretical and numerical models of the coupled electro-chemical, thermal, and fluid processes in (SOFC) have been developed, and multiple papers have been published on the subject. The first modeling efforts were highly simplified and limited to predicting average cell values such as voltage, current density, and temperature in isothermal cells-which side steps the issue of thermal radiation altogether. More detailed, non-isothermal numerical models, began to appear in the early 1990s, and building on these pioneering works, Hartvigsen et al. [2] were the first to consider surface-to-surface radiation exchange in the flow (air and fuel supply) channels, and finally conclude with the analysis of stack level thermal radiation effects including high temperature thermal insulation and overall stack thermal management.

## Heat transfer in solid oxide fuel cells

Solid oxide fuel cells are solid state devices that convert the chemical potential in fuel and oxygen directly into electricity by taking advantage of the capacity of zirconia to conduct oxygen ions. A cell consists of three layers, as shown in **Figure. 1**: an anode layer typically composed of porous cermet (most often a combination of Ni and yittria-stabilized zirconia, YSZ), a solid electrolyte layer composed of YSZ, and a cathode usually made up of porous strontium-doped lanthanum magnate (LaMnO3),

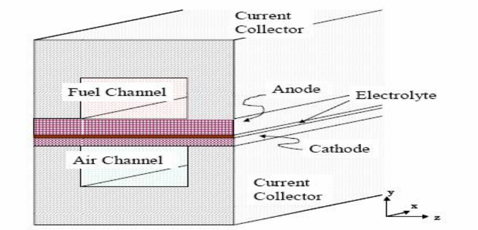


*Fig.1.Shematic of the planar, anode-supported unit cell model of SOFC.*

a perovskite. These layers are arranged in planar or tubular configurations to create a single cell (this paper focuses on the planar arrangement.) One of the three layers provides structural support and is consequently thicker than the other two; the

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majority of SOFC are anode- or cathode-supported, since a thin electrolyte layer facilitates ion transfer between the electrodes. Air channels provide oxygen to the cathode, while fuel channels provide hydrogen and remove water vapor from the anode. Individual SOFC cells are encased in a stainless-steel or ceramic interconnect and then connected electrically to
form a stack [3].
Electricity is generated by the electrochemical reactions at the electrode–electrolyte interfaces, as shown in **Figure. 2**.(The porosity of the electrodes enables the reactants and
products to migrate between the electrode surface and the electrode–electrolyte interface.) At the cathode–electrolyte interface, oxygen is combined with electrons supplied by the interconnect to yield oxygen ions.

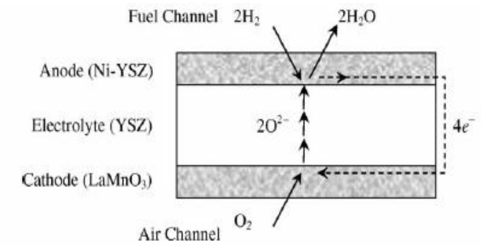


*Fig. 2 Schematic of electrochemical reactions.*

## Calculation of cell temperature

The cell temperature distribution in the whole cell stack is calculated by solving the three dimensional time dependent heat conduction equation using the finite volume method.

The heat conduction equation can generally be written in the following form.

$$\frac{\partial}{\partial x}(\lambda x.\frac{\partial T}{\partial x}) + \frac{\partial}{\partial y}(\lambda y.\frac{\partial T}{\partial y}) + \frac{\partial}{\partial z}(\lambda z.\frac{\partial T}{\partial z}) + S_T = \rho Cp\frac{\partial T}{\partial t} \qquad (1)$$

where q is the mass density, c is the specific heat, k is the thermal conductivity and S is the heat generation per unit volume.
In the simulation code, the effect of heat convection with the gas flow and heat transfer between the solid and gas are taken into account by including them in the heat generation term S, and the following equations are used as this term for the electrolyte/electrodes, fuel side interconnector and air side interconnector in each single cell plate [4].

## Modeling methology

### Radiative properties

The prediction of radiation heat transfer in semi-transparent materials requires the knowledge of radiative properties, namely, the absorption coefficient and the scattering coefficient of the material. The state-of-the-art SOFCs uses yttria-stabilized-zirconia (YSZ) as the electrolyte, strontium-doped LaMnO3 (LSM) as the cathode and nickel doped YSZ as the anode. The radiative properties of a material are usually a function of wavelength, and the spectral region of interest for thermal radiation can be obtained from the Planck's law for blackbody emissive power,

$$E_{b\lambda} = \frac{2\pi h c_0^2}{n^2\lambda^5\left[e^{hc_0/n\lambda kt} - 1\right]} \qquad (2)$$

where $E_B\lambda$ is the spectral emissive power of a black body, $h$ the Planck constant, $c0$ the speed of light in vacuum, $k$ the Boltzmann's constant, $T$ the temperature and $n$ the refractive index of the medium. The variation of the black body **Fig.3**.

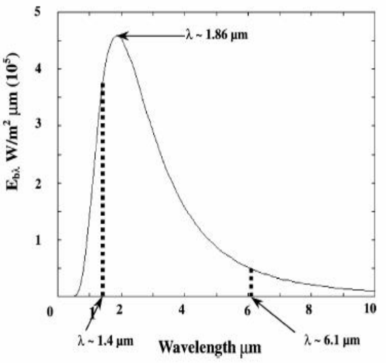


*Fig.3. Spectral black body emissive power for n=1.6, T=973 k, of radiative energy contained within 1.4 $\mu m$.*

Spectral black body emissive power for $n = 1.6$, $T = 973$ K, 80% of radiative energy contained within

1.4 $\mu m < \lambda < 6.1\ \mu m$. emissive power with wavelength for $n = 1.8$ and $T = 973$K is shown in **Fig. 3** and corresponds to SOFC operating conditions. It can be shown that 80% of the radiative emissive power is contained within the infrared spectral region 1.4 $\mu m < \lambda < 6.1\ \mu m$ with the maximum at $\lambda \approx 1.86\ \mu m$.
**Fig. 4** shows the radiative properties of YSZ obtained with a 500 $\mu m$ thick sample. Clearly, a gray body approximation can be used in the radiation modeling of YSZ, as the transmittance is found to be independent of wavelength in the region of interest (1.4 $\mu m < \lambda < 6.1\ \mu m$). The absorption coefficient of YSZ electrolyte is obtained from.

$$T_r = e^{-kL} \qquad (3)$$

where Tr is the transmitivity, $\kappa$ the absorption coefficient and $L$ the thickness of the medium. The optical properties of the cathode are obtained from the imaginary part of dielectric constant spectra and the optical conductivity spectra [8] of

La1−xSrxMnO3. The optical thickness ($\tau = \kappa . L$) of the fuel cell electrolyte

*Fig.4.optical properties of a 500 $\mu m$ thick YSZ sample*

and cathode, based on the dimensions , were found to be 0.25 and 104, respectively. The optical properties of the anode material are complicated to calculate as they depend on the distribution of the nickel cermets in YSZ. The presence of Nickel doping makes the anode highly absorbing to thermal radiation, thus resulting in optically thick ($\tau \succ\succ 1$) behavior. Therefore, for lack of experimental data, the absorption coefficient of the anode is assumed to be the same as that of the cathode. Also, the radiation scattering in electrodes is neglected, although this assumption is questionable considering the porous nature of electrodes.

**Numerical conditions**

The length, width and height of the single cell plate in the analyzed cell stack are selected as 200 mm x 200 mm x 3.2 mm. An almost cube shaped cell stack is assumed, and the number of single cells piled in the stack is selected as 70mm. The height of the total cell stack is 224 mm in this case. These and the other dimensions of the analyzed cell stack are summarized in Table 1. As the cell materials, YSZ, Ni/YSZ, La0.8Sr0.2MnO3 and LaCr0.9Mg 0.1O3 are selected for the electrolyte, anode, cathode and interconnector, respectively.

**Table.1** dimensions of calculated cell stack

| | |
|---|---|
| Number of single cells in stack | 70 |
| Cell stack length | 200 mm |
| Cell stack width | 200 mm |
| Cell stack height | 224 mm |
| Single cell height | 3.2 mm |
| Channel pitch | 4 mm |
| Channel width | 2 mm |
| Channel height | 1 mm |
| Electrolyte thickness | 0.1 mm |
| Fuel electrode thickness | 0.05 mm |
| Air electrode thickness | 0.05 mm |

**Effect of radiation heat transfer**

To make clear the effect of radiation heat transfer from and to the cell stack surfaces, numerical simulations of the co-flow type cell are performed in the cases of various ambient temperatures between 1073 K (800 C°) and 1273 K (1000 C°), and the calculation results in steady state are compared with their corresponding results obtained by the authors' previous simulation codes for the one channel region and for the single cell plate.

**Coupling of radiation to overall energy conservation**

In the optically thick case of SOFC electrodes, augmentation of total heat flux due to thermal radiation is accounted for by using an effective thermal conductivity, given by the sum of the intrinsic thermal conductivity of the medium and the Rosseland radiative "conductivity" defined as [5],

$$k_R = \frac{16n^2\sigma T^3}{3\beta_R} \qquad (4)$$

where $T$ is the absolute local temperature (K), $\sigma$ the Stefan–Boltzmann constant($5.67\times10{-8}Wm^{-2} K^4$), $n$ the refractive index of the medium, and $\beta$ is the spectrally averaged **Rosseland**-mean extinction coefficient of the medium.

$$\frac{1}{\beta} = \frac{\pi}{4\sigma T^3}\int_0^\infty \frac{1}{\beta_n}\frac{dI_{bn}}{dT}dn \quad (n = \frac{1}{\lambda} \text{ is the wave number}) \qquad (5)$$

Radiation is thus coupled to overall energy conservation as,

$$q_{total}(z) = q_{cond}(z) + q_{rad}(z) = -k\frac{dT}{dz} - k_r\frac{dT}{dz} = -(k + \frac{16n^2\sigma T^3}{3\beta_r})\frac{dT}{dz} \qquad (6)$$

However, caution must be exercised because this approximation does not perform well near boundaries where even an optically very thick medium is locally optically thin. A comparison of the radiative conductivity to the intrinsic thermal conductivity indicates the relative magnitudes of radiation
and conduction heat fluxes. As shown in the analysis by Murthy and **Fedorov** [5], the radiative conductivity of the electrodes is much smaller than the overall thermal conductivity, implying that radiation in the electrodes could safely be neglected [6].

**Radiative model approximation**

This limitation can be overcome by coupling the diffusion model with the Schuster–Schwartz child two-flux approximation for locally optically thin regions near the boundaries of the overall optically thick medium.

### Schuster–Schwartz child two-flux approximation

Schuster–Schwartz child or the two-flux approximation provides a simple solution method for one-dimensional radiation in a thin, plane-parallel slab. In this method, the radiative intensity is assumed to be an isotropic function of the propagation direction, but different, over the upper and lower hemisphere. For a gray, non-scattering medium confined between two isothermal, parallel black plates at temperatures *T*(**top**) and *T*(**bottom**) and separated by a distance *L*, the two-flux model gives the radiative heat flux as [6].

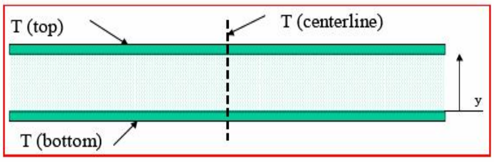


*Fig.3 Schematic of two-dimensional model validation of Schuster- two flux radiation model.*

$$q_R(z) = C_1 e^{2kz} + C_2 e^{-2kz} \qquad (7)$$

where:

$$C_1 = -\sigma\left(T_{top}^4 - T^4\right)e^{-2kl} \quad ,$$

$$C_2 = \sigma\left(T_{bottom}^4 - T^4\right) \qquad (8)$$

The radiation transport thus calculated can be coupled with the overall energy conservation by introducing the divergence of the radiative heat flux as a (negative) source term in the energy equation. **Figure. 3** shows a schematic of a two-dimensional enclosure used for validating the Schuster Schwartz child radiation model. The bottom and top walls of the enclosure were maintained at constant temperatures of 1000 K and 2000 K, respectively, with the sidewalls being adiabatic. Studies were carried out for different optical thickness by varying the absorption coefficient of the enclosed medium. **Figure. 4** shows the temperature variation across the thickness of (0.001), (0.01), ( 0.1), (1). of the liquid medium as predicted by the two-flux approximation, Excellent agreement in temperature predictions is seen for extremely optically thin materials [7]. Large deviations are observed with increase in optical thickness with the Schuster–Schwartz child approximation failing for optical thickness greater than one.

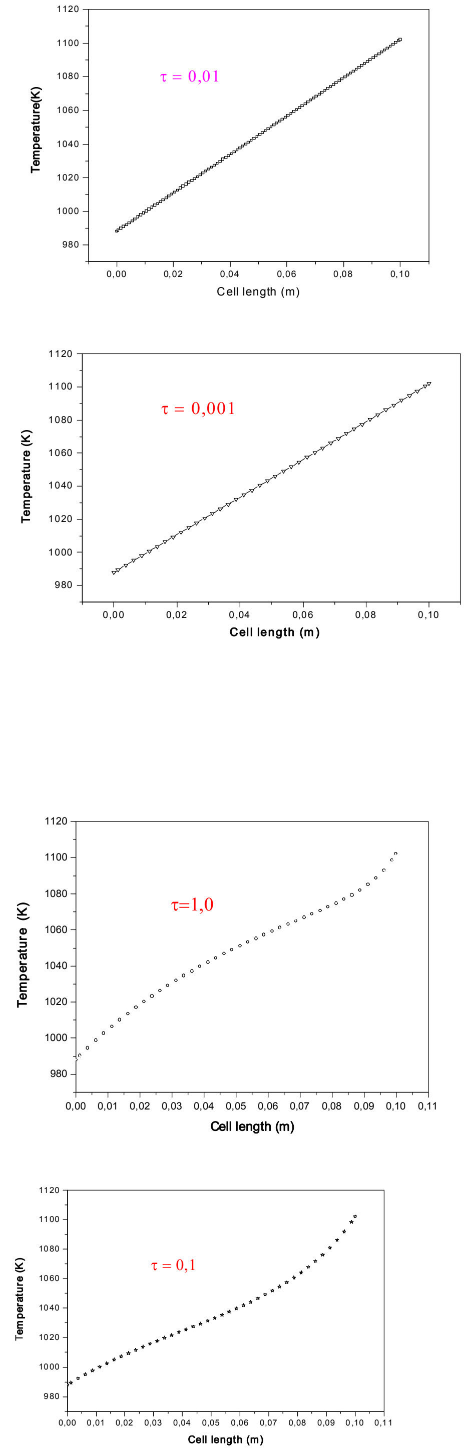


*Fig.4 scheme for various optical thicknesses with radiation term*

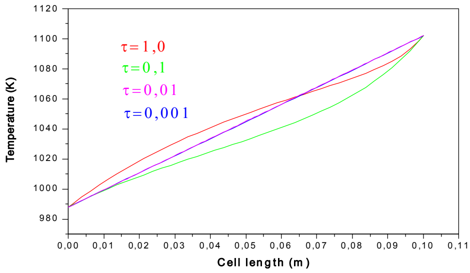


*Fig.5 comparison for different optical thickness.*

## Model Results and Analysis

Finally, although our results indicate that radiation within the electrolyte has little effect for the geometry and operating conditions used in this study, it cannot, in general, simply be neglected. For example, during the start-up and shut-down of the SOFC, one could develop significant temperature gradients across even very thin electrolyte leading to potentially significant radiation effects. The major concern on whether or not to include the radiation analysis into the SOFC thermal model lies in the tremendous increase in computational costs associated with inclusion of radiation. This demands development of computationally efficient techniques for treatment of radiative heat transfer.

## Conclusions

Based on the rigorous analysis of radiation heat transfer in a single unit SOFC, the following conclusions can be drawn:

(i) Though commonly neglected, radiation heat transfer effects are significant and need to be accounted for
accurate prediction of the temperature field and the fuel cell output voltage.

(ii) The computationally expensive discrete ordinate method can be replaced by a simplified radiation sub-model, which is based on Rosseland diffusion approximation for the optically thick electrodes and the Schuster–Schwartzchild two-flux method for the optically thin electrolyte.

## NOMENCLATURE

$I_\lambda$ Spectral intensity of radiation ( $w\,m^{-2}\ \mu m^{-1}$ ).
N refractive index of medium.
$q_r$ total radiative heat flux ( $w\,m^{-2}\ \mu m^{-1}$ ).
$q_{con}$ total conduction heat flux ( $w\,m^{-2}\ \mu m^{-1}$ ).
T absolute temperature (K).
K absorption coefficient ($m^{-1}$).
h Plank's constant(J.s).
$c_0$ speed of light in vacuum (m/s).
$T_r$ transmittance.

## Greek letters:

$\sigma$ Stefan-Boltzmann constant (j $K^{-4}m^{-2}s^{-1}$).
$\beta_r$ Rosseland-mean extinction coefficient ($m^{-1}$).
$\lambda$ Wavelength ( $\mu m$).
$\zeta_l$ optical thicknesses of electrolyte (m).
$\alpha$ thermal diffusivity ($m^2.s^{-1}$).
YSZ yttria-stablized zirconia.
SOFC Solid Oxide Fuel Cells.

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