Extraction of organic and inorganic solar cell device parameters using vertical optimization and Lambert W-function

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Abstract— A numerical procedure for the simultaneous determination of the various parameters of solar cells and Schottky diodes from the measurement of a single current-voltage (I-V) curve has been re-examined. The technique is based on vertical optimization method and the exact explicit solution of the I-V relation expressed by Lambert W-function. Tests reveal that the proposed method is very fast and efficient for extracting the five illuminated solar cells parameters. Experimental results are presented and compared to values of some parameters extracted by other previously published methods both on organic and inorganic solar cells. Furthermore, the influence of errors existing during I-V measurements on these currents has also been investigated. The proposed method is fast, accurate and can be applied for a commercial, a module and an organic cell

Keywords: extraction, Solar cells, Lambert W-function, Vertical optimization.

I. Introduction

In the process of characterization of photovoltaic devices and in order to evaluate or predict their performances it is important to extract the corresponding electrical parameters by a fast and accurate method. These parameters are usually the reverse saturation current (I_0), the series resistance (R_s), the ideality factor (n), the shunt resistance (R_{sh}) and the photocurrent (I_{ph}). Several methods for solar cell parameter extraction have been proposed to yield values for only some or all the parameters using the *I–V* characteristics [1-30].

The methods of parameters extraction can be classified into two main groups: direct/algebraic manipulation estimation [2-4, 7, 11, 13, 14, 18-20, 22, 28-30] and optimization methods [1, 5, 6, 10, 12, 15-17, 21, 23, 24, 26, 27]. The advantages of using direct estimation or algebraic manipulation are increased speed and relative simplicity compared to optimization methods. However, the error in the final extracted parameters is the main drawback of direct estimation methods. The methods which use algebraic manipulation are sensitive to noise or random errors during measurements. Optimization methods remain attractive and in order to extract parameters over the whole bias range (also reverse), there is no other alternative to such methods. Advantages and disadvantages of some proposed methods and comparative studies have been dealt with in previous papers [9, 25, 32-35].

Several works have appeared recently which discuss again the problem of solar cell extraction parameters [31, 36-40]. These methods make use of what is known as the *Lambert W function.* The history, mathematical developments and properties of Lambert W are reviewed in Corless *et al* [41]. Applications to physics can be found in [42-45]. A careful search in literature reveals that the Lambert W function has been used to find the exact explicit solution I-V relation in the case of pn junction [47], bipolar transistor [48], non-ideal diodes [49] as well as the solution of the channel surface potential as function of voltage in MOSFETs devices [50].

The purpose of this paper is to re-examine the vertical optimization method (VOM) approach for solar cells extraction based on computer programs making use of the Lambert W-function resulting in a new, very fast and accurate extraction method. This method has been successfully applied to extract Schottky diode parameters with [51] and without shunt conductance [52]. In this work we extend its application to evaluate the five illuminated solar cell parameters and using the explicit current voltage relation.

This quick and accurate method has been applied for a commercial, a module and an organic cell. Finally, the validity of the extracted parameters has been checked by comparing our results with values obtained by alternative methods in previously published works.

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II. THEORY AND PROPOSED METHOD

The current–voltage (I-V) characteristics of a realistic illuminated solar cell with series resistance (R_s) and a shunt resistance (R_{sh}) (or equivalently a parallel conductance G_{sh}) for a single exponential model is usually given by:

$$I = I_{ph} - I_0\left[\exp\left(\frac{(V+R_S I)}{nV_{th}}\right)-1\right]-\frac{(V+R_S I)}{R_{sh}} \qquad (1)$$

Here V_{th}, I_{ph}, I_0 and n are the thermal voltage, the photocurrent, the reverse saturation current and the ideality factor respectively. The I-V characteristics can be represented by a two diode model. The single diode model is however the most popular used model for solar cell parameters which are under normal operating conditions [13, 14].

Given N experimental data I_{ej}-V_{ej} (j=1..N) representing the direct and/or the reverse region of the I-V characteristic, the problem that we have to solve is to extract the five lumped parameters in order to have the best fit between the experimental current and the theoretical one (I_{th}) generated by Eq.1. We evaluate the parameters by using an optimization method in order to minimize an objective function S:

$$S = \sum_{j=1}^{N}\left(\frac{I_{ej}-I_{th\,j}}{I_{thj}}\right)^2 \qquad (2)$$

We make use the quadratic relative difference between the experimental and the simulated values of the current because the I-V characteristic of solar cells is usually measured over more than four decades on the current axis.

In the VOM, the minimum of S is found by the resolution of the set of non linear equations:

$$\frac{\partial S}{\partial P_i} = 0 \qquad (3)$$

Where P_i is one of the five free lumped parameters which are used for fitting the I-V curve.

Performing derivative in (3), we obtain:

$$\sum_{j=1}^{N}\frac{I_{ej}-I_{th\,j}}{I_{thj}}\frac{I_{ej}}{I_{thj}^{\,2}}\frac{\partial I_{thj}}{\partial P} = 0 \qquad (4)$$

Instated of using the implicit expression of I_{th} given by equation (1) and its calculated derivatives (as generally done), an explicit solution of eqn. (1) is given by:

$$I = -\frac{V-R_{sh}(I_0+I_{ph})}{R_s+R_{sh}} - \frac{nV_{th}}{R_s}W\left[\frac{R_s I_0}{nV_{th}(1+R_s/R_{sh})}\exp\left(\frac{V+R_s(I_0+I_{ph})}{nV_{th}(1+R_s/R_{sh})}\right)\right] \qquad (5)$$

Where W denotes the principal branch W_0 of the Lambert W function. The first derivative which satisfies:

$$\frac{\partial W(x)}{\partial x} = \frac{W}{1+W}\frac{1}{x} \qquad (6)$$

is also useful in several calculations when evaluating derivatives in (4).

The solution of the set of equations given by eqn. (4) can be found by Newton’s method. It is well known that the main advantage of Newton's Method is its rapid convergence but the iteration process may diverge if a good initial set of parameters values can't be found.

It has been found that the problem of undesired oscillation and eventual overflow in the objective function is eliminated (reduced) when using the explicit form of the I-V expression given by the Lambert W-function. Moreover, the region of convergence of the five parameters that have been to be initialized is enlarged which ensures convergence to solution.

III. RESULTS

We present the results of the application of the proposed method on a 57mm diameter commercial silicon solar cell, a solar module in which 36 polycrystalline silicon solar cells are connected in series and an organic solar cell [53].The physical parameters for the given cell or the module are first extracted from measurements of the I-V characteristics using VOM.

Prior to this, the method was tested using simulated data sets from I-V curves because this is the only way to be certain that the final results is an ideal fit, and allows the physical and mathematical problems involved in the fitting process to be decoupled. In addition random generated noise has also been added to the simulated I-V curves to account for possible errors in measurements.

Data were generated using eqn. (1) and were used as inputs to test the accuracy, efficiency and robustness of the proposed method. Several simulations were performed using different conditions.

The method enables one to extract parameters using the whole bias range (also reverse or only forward/reverse) of the diode characteristics. The method is insensitive to the number of measurement data as long as it is greater than 30. Using the number of points above 100 the method always found the values of the parameters with accuracy better than 1%. The total number of data points depends on their distribution as well and increasing the number of points well distributed improves the accuracy in most simulations.

We have also tested the dependence of the method on the initial values by starting the algorithm with following initial conditions 0.5x, 0.9x, 1.5x and 2x of the correct solution. In very close proximity to the correct solution (0.5x, 0.9x and 1.5x) the method was able to recover the correct solution. Otherwise, is always diverge or stopped to local minimum.

Random measurement errors have been added to the generated ideal curve. It has been observed that the extracted n, R_s, G_{sh} and I_{ph} have very small absolute relative errors when the measurements are within the tolerance of a typical experimental setup (< 5%). The accuracy of the extracted parameters becomes questionable when the level of noise is very high (>10%).

The method is then applied to measured I–V data of different real solar cells. Inorganic and organic cells are considered in this work. It is obviously clear that we will not

TABLE I
OPTIMAL INITIAL ORGANIC AND INORGANIC SOLAR CELLS PARAMETERS USING THE GA METHOD

	Cell (33°C)		Module (45°C)		Organic cell (27°C)	
Para-meter	Initial specified Ranges	GA	Initial specified Ranges	GA	Initial specified Ranges	GA
R_s (Ω)	[0.01, 0.6]	0.0454	[1, 2]	1.58	[0.001, 0.005]	0.0047
G_{sh} (Ω^{-1})	[0.01, 0.5]	0.0297	[0.001, 0.01]	0.00144	[0.1, 10]	0.72
n	[1, 2]	1.5	[40,60]	48.5	[1, 5]	3.2
I_s (mA)	[0, 1]	0.325	[1, 5]	1.05	[1, 20](nA/cm^2)	12.1
I_{ph} (A)	[2, 10]	0.762	[1, 10]	1.2	[0.1, 1](mA/cm^2)	0.81

use the ideal parameters as inputs. An initial values were estimated using available techniques [22, 27]. In this work a slightly modified genetic algorithm (GA) procedure has been applied as described in more detail in a previous work [27]. In fact, instead of using the implicit form for the current voltage diode equation, the explicit form of the I-V relation given by the Lambert W-function (eqn. 5) has also been used.

The most remarkable feature in using the GA method lies in its convergence to reasonably good solutions despite the wide ranges to determine approximate starting values for these parameters.

The GA method in this respect is advantageous especially in situations where it is difficult to anticipate on an initial guess for the desired solution.

The best fitness parameters are shown in Table1 along with the initial range specified for each parameter.

The extracted parameters values of the real inorganic and organic cells and the calculated characteristic ones (I_m, V_m, FF) are given in Table 2 which includes also the values of parameters obtained using other methods for the sake of comparison. In order to do quantitative comparison we have calculated the standard deviation σ for each method in order to test the quality of the fit to the experimental data:

$$\sigma = \sqrt{\frac{1}{N}\sum_{j=0}^{N-1}\left(\frac{I_{thj}}{I_{ej}} - 1\right)^2} \qquad (7)$$

Satisfactory agreement is obtained for most of the extracted parameters.

As can be seen from the tables the VOM gives the best deviation both for the cell and the module. The results show also that the use of VOM for solar cells parameters extraction significantly decreases the error in the extracted values and hence improve the accuracy of the determined parameters. Is worth noting that the parameters eventually determined, particularly the ideality factor, in the case of the module will lose somewhat their physical meanings. The model parameters are not well related to the physical parameters due to the series connection of the cells to form modules.

Figs. 1–2 show the plots of I–V experimental characteristics (closed circles) and the fitted curves (solid lines) derived from Eq. (1) with the parameters shown in Table 2 for the silicon solar cells and the organic solar cell. Very good agreement is observed for the whole bias range.

The interesting points with the procedure described herein is the fact that it has been successfully applied to experimental I–V characteristics of different types of solar cells from inorganic to organic solar cells with completely different physical characteristics and under different temperatures.

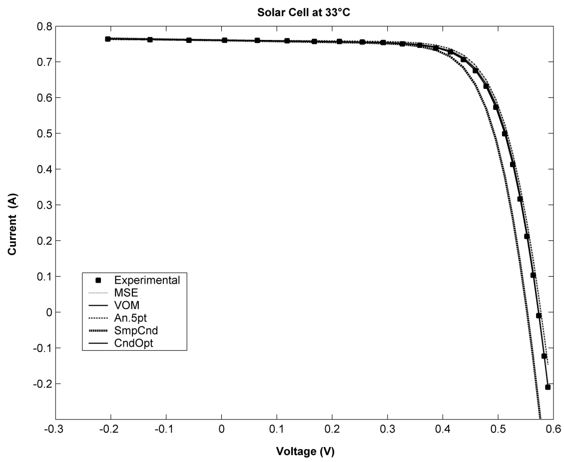

Fig. 1. I-V characteristics of solar cell

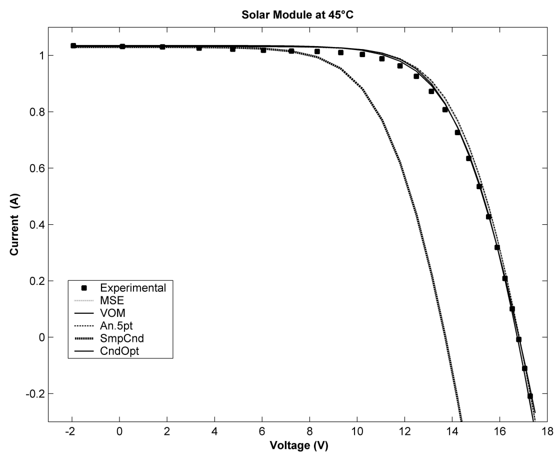

Fig. 3. I-V characteristics of solar module

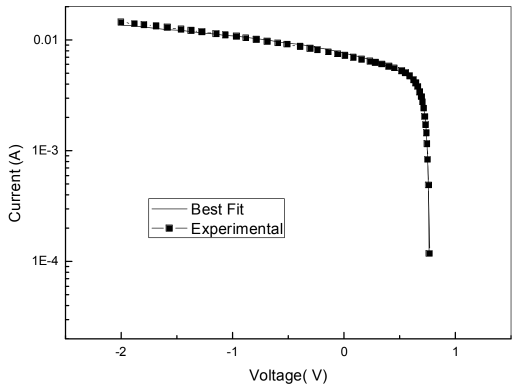

Fig. 3. I-V characteristics of plastic solar cell.

TABLE II

OPTIMAL INITIAL ORGANIC AND INORGANIC SOLAR CELLS PARAMETERS USING THE GA METHOD

Para-meters	MSE [3]	An.5pt [3]	CndOpt [25]	SmpCnd [25]	VOM our Work
			Cell at 33°C		
G_p (Ω^{-1})	0.0186	0.0094	0.0202	0.02386	0.0200
R_s (Ω)	0.0364	0.0422	0.0364	0.0385	0.0370
N	1.4837	1.4513	1.5039	1.456	1.5430
I_s (mA)	0.3223	0.2417	0.4039	0.046	0.5923
I_{ph} (A)	0.7608	0.7606	0.7608	0.7603	0.7605
σ (%)	1.4743	2.1549	1.0359	8.4973	0.6790
I_m (A)	0.6894				0.6860
V_m (A)	0.4507				0.4471
I_{sc} (A)	0.7603				0.7599
V_{oc} (V)	0.5728				0.5717
FF	0.7135				0.7060
			Module at 45°C		
G_p (Ω^{-1})	0.00182	0.00145	0.005	0.00145	0.00195
R_s (Ω)	1.2057	1.2226	1.146	1.2293	1.5900
N	48.450	47.533	51.32	48.93	43.292
I_s (mA)	3.2876	2.5908	6.77	46	1.0453
I_{ph} (A)	1.0318	1.0320	1.035	1.030	1.0318
σ (%)	1.6378	1.8428	1.1973	53.2723	1.2534
I_m (A)	0.912				0.9071
V_m (A)	12.649				12.142
I_{sc} (A)	1.030				1.0286
V_{oc} (V)	16.778				16.337
FF	0.668				0.6554

Organic Cell at 27°C

Para-meters	Co-C funct [31]	Optim. Met.[31]	5 point Meth[31]	App. Meth [46]	VOM our Work
G_p (Ω^{-1})	0.0057	0.0049	0.0202	0.0057	0.0036
R_s (Ω)	8.59	6.21	0.8	1.02	0.63
N	2.31	2.32	3.45	2.2	2.78
I_s (nA/cm^2)	13.6	13.6	9.57	6.35	11.6
I_{ph} (mA/cm^2)	0.760	0.7606	0.7608	0.7603	0.728
σ (%)	-	-	-	-	0.6790

IV. CONCLUSION

A numerical procedure is described herein for the extraction of the illuminated solar cell parameters. The methods enable us to extract the cell parameters for diodes with high ideality factor. The VOM method is quite accurate, fast and does not require any kind of graphs for the analysis of the I-V characteristics. The method uses the whole bias range (also reverse) of the characteristics for parameter extraction. However the method needs a good initial guess. GAs method is advantageous especially in situations where it is difficult to anticipate an initial guess for the desired solution. In conclusion one can use standard methods in order to obtain approximate values for the solar cell parameters. GAs method is then used to improve the accuracy of the extracted values. The parameters obtained are then used as initial guesses in VOM method. The procedure can be extended to two diodes model of solar cells.

ACKNOWLEDGMENT

The authors gratefully acknowledge the support of Dr A. Sellai from SQU University of OMAN.

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