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Using Robust sliding mode controller to improving the aerodynamic performance of a gas turbine

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Abstract- **Gas turbines are complex processes characterized by the instability and uncertainty of various sources. The range of useful operating in an axial compressor which is part of a turbine gas is limited by aerodynamic instabilities that are pumping and rotating stall. These hazardous phenomena limit compression systems performance and can cause mechanical damages. Furthermore speed transitions, which can lead to temporary stall development and pressure drop at the output, degrade the effective operation of compressors and consequently gas turbines. Several mathematical models have been developed for explain the operation of a gas turbine. These models give good understanding nonlinearities, providing an effective way to develop control strategies to increase the operating range and improve performance of a gas turbine. Due to not precise knowledge of the compressor map and not full-state feedback of these models, a robust nonlinear control method based on feedback linearization is applied to tackle this open control problem.**

Keywords— ***Gas turbine, Axial compressor , surge, rotating stall, sliding mode control, robust non linear control.***

I. Introduction

The development experienced by civil or military aviation, the growth of the industry processes as well as the field of the generation of one energy are related to the development an essential element which is the gas turbine. The latter is a machine that produces mechanical energy and high velocity exhaust gases and high temperature. The mechanical energy is used to drive a compressor, fan, etc.

The gas turbine is however subject to nonlinear phenomena of different nature: aerodynamic (pump and rotating stall), aero-elasticity (the float) and combustion that do not allow proper operation [1]. In this work, we will focus on the aerodynamic non linearities. Gas turbine suffer from two types of instabilities which limit their efficiency and performance: rotating stall and surge. These phenomena are closely related. Rotating stall is a non-axisymmetric perturbation that travels around the annulus of the compressor while surge is a large axial oscillation of the flow [1].

The low order model of Moore and Greitzer for the post stall transients of axial compression systems has been used extensively in stall/surge analysis and control. In the original work of Moore and Greitzer the compressor speed is assumed constant. If the equilibrium of the compression systems is located to the left of the surge line which passes through the local maxima of the compressor characteristic, the flow becomes unstable. Dependent on certain system parameters and as be demonstrated in [2], the compressor speed, the instability can take the form of rotating stall, surge or both. This model has been successfully applied to a wide variety of stability and control problems.

Moore and Greitzer model has the following advantages: it captures most nonlinear and operational effects. It is low order and physical rather than computational [3]. In 1997, Gravdahl and Egeland [1] derived a similar model and investigated surge and speed control. However, these models were both developed for centrifugal machines, and did not include rotating stall as a state. For the first time, the model developed by Gravdahl for axial compressors considered the B-parameter (proportional to the speed of the compressor) as a state and included higher harmonics of rotating stall as well [3].

This new high order extension of Moore-Greitzer model not only shows the previous qualitative behavior such as surge and stall development but also introduces some novel phenomena as a direct consequence of adding the new state to the model. These phenomena can exclusively be described by the non-constant speed model [1]. Gravdahl [3] initially demonstrated the temporary development of rotating stall at an operating point far from surge line because of speed variations. Model simulations showed that amplitudes of rotating stall harmonics temporarily increase while the machine is accelerating, but are quickly damped out as desired speed is reached [4]. Output pressure also drops during speed transitions.

Contrary to Gravdahl's non-constant speed model, Moore-Greitzer original model does not imply any rotating stall development, since the working point is situated by an adequate margin to surge line. This temporary stall development and pressure drop can cause trouble for the normal operation of turbo machines [4]. Furthermore, including model uncertainties (the precise estimation of model parameters, especially in the unstable zone, being difficult) and external perturbations make the problem even more challenging [10]. Finally, the squared amplitude of stall modes used as state variables are experimentally difficult to measure and full-state feedback cannot be considered in control design. In this work, throttle and close-coupled valve (CCV) actuation are used to guarantee the stability, and a drive torque is applied to increase the speed of the rotor. CCV is considered to be one of the most promising actuation methods [5]. To develop the controller, the amplitude of rotating stall, which cannot be measured but is one of the state variables in Gravdahl's model, is considered as a disturbance.

This assumption being supported by the proof of perturbations boundedness greatly simplifies the design. Furthermore, the control scheme does not require an accurate knowledge of model parameters. Simulation results corroborate analytical developments and demonstrate the disturbance rejection and the global ultimate boundedness of state variables which leads to surge and rotating stall control.

II. Gravdahl-England Model Developement

This is an extension of the Moore-Greitzer model, where appropriate considering the compressor speed variation. It is also a multi-mode model to a better understanding of aerodynamic non linearities: the rotating stall and pumping. This model contains the parameter B Greitzer as a state. It incorporates more dynamic spool (shaft).

In this section we review the development of Gravdahl's model [2] for variable speed axial compressors in order to render the paper reasonably self-contained. The compression system consists of an inlet duct, inlet guide vanes IGV, variable speed axial compressor, exit duct, plenum volume and throttle (Figure 1). Throttle can be regarded as a simplified model of a turbine.

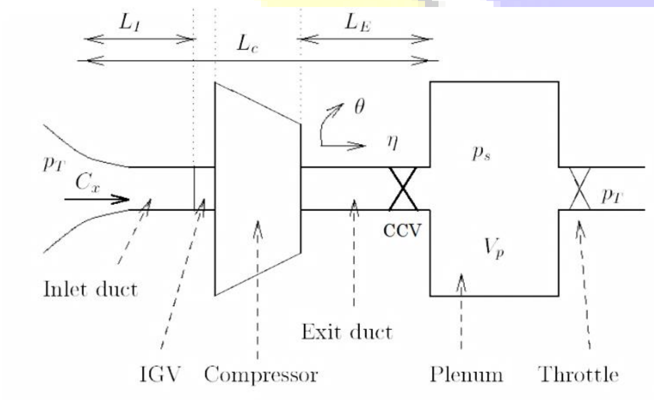

Fig.1: Schematic of compressor showing non dimension lengths [2]

The Gravdhal final model with Closed coupled Valve, taking en consideration the spool dynamics:

$$\frac{d\Phi}{d\xi} = \frac{H}{l_c(U)}\left[-\frac{\Psi-\psi_{c0}}{H} + 1 + \frac{3}{2}\left(\frac{\Phi}{W}-1\right)\left(1-\frac{J}{2}\right) - \frac{1}{2}\left(\frac{\Phi}{W}-1\right)^3 - \frac{1}{{\gamma_v}^2}\left(\frac{\Phi^2}{H}+\frac{W^2 J}{2H}\right) - \frac{U_d \Lambda_1 \Gamma l_E \Phi}{bH}\right] \quad (1)$$

$$\frac{dJ_n}{d\xi} = J_n\left[1-\left(\frac{\Phi}{W}-1\right)^2 - \frac{J_n}{4} - \frac{\mu n^2 W}{3aH} - \frac{2U_d\Lambda_1\Gamma(m-1)W}{3Hbn} - \frac{4\Phi W}{3H{\gamma_v}^2}\right]\frac{3aHn}{(n-m_U(U)a)W} \quad (2)$$

$$\frac{d\Psi}{d\xi} = \frac{\Lambda_2 b}{U}(\Phi-\Phi_T) - 2\Lambda_1\Gamma\frac{U}{b}\Psi \quad (3)$$

$$\frac{dU}{d\xi} = \Lambda_1\Gamma\frac{U^2}{b} \quad (4)$$

The precedent equations (1-4) introduce the new state space model in which higher harmonics and the effect of viscosity are included. In these equations W, H and ψ_{c0} are parameters defining the shape of the compressor characteristic map (Figure 2). a and b are constants (see [7, 8] for further details.). All distances are non-dimensionalized with respect to the mean compressor radius R. $l_E = {}^{L_E}/_R$ and $l_C = {}^{L_C}/_R$ where the lengths l_E and l_C are depicted in Figure 1. In what follows, a normalized time $\xi = U_d * t/R$ is also used to write the dynamics where U_d is the desired constant velocity of the wheel.

The turbine torque Γ_t and the compressor torque Γ_c are non-dimensionalized

$$\Gamma = \Gamma_t - \Gamma_c \quad (5)$$

The compressor torque is given by

$$\Gamma_c = -\Phi^2(\tan\beta_1 - \tan\beta_2) \quad (6)$$

β_1 and β_2 are constant blade angles at rotor entrance and exit respectively [4]. The term $(\tan\beta_1 - \tan\beta_2)$ is constant.

The relation between m_U and the compressor speed U is:

$$m_B = (m-1)\frac{U_d}{U} - 1 \quad (7)$$

Compression system characteristic (compressor map) $\Psi_c(\phi)$ represents the relation between pressure rise at the output of the compressor and mass flow expressed as a nonlinear curve.

$$\Psi_c(\Phi) = \psi_{co} + H * \left[1 + \frac{3}{2}\left(\frac{\Phi}{W}-1\right) - \frac{1}{2}\left(\frac{\Phi}{W}-1\right)^3\right] \quad (8)$$

Note that, throttle characteristic is considered as:

$$\phi_T = \gamma_T\sqrt{\Psi} \quad (9)$$

large γ_T implies an open throttle and small γ_T means a closed throttle, and γ_v is closed coupled valve gain.

At equilibrium, we have:

$$\dot{\Phi} = \dot{\Psi} = 0 \quad (10)$$

$$\dot{J} = J\left(1 - \left(\frac{\Phi}{W} - 1\right)^2 - \frac{J}{4}\right) = 0 \quad (11)$$

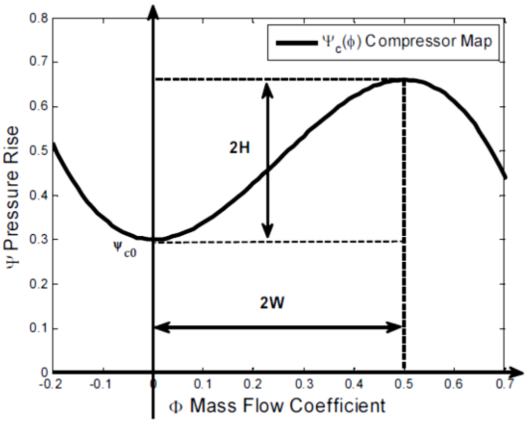

Fig. 2: Compressor Map W, H and ψ_{c0} [4]

which lead to two equilibria. The first one, $J_{e1} = 0$, corresponds to the compressor being in its active operating point (OP). The second one, $J_{e2} = 4\left(1 - \left(\frac{\Phi}{W} - 1\right)^2 - \frac{J}{4}\right) > 0$, corresponds to the system being in a fully developed rotating stall. For the second J_{e2}, one can obtain the stall characteristic $\Psi_s(\Phi)$of the compressor [4].

$$\Psi_s(\Phi) = \psi_{co} + \mathrm{H} * \left[1 - \frac{3}{2}\left(\frac{\Phi}{W} - 1\right) - \frac{5}{2}\left(\frac{\Phi}{W} - 1\right)^3\right] \quad (12)$$

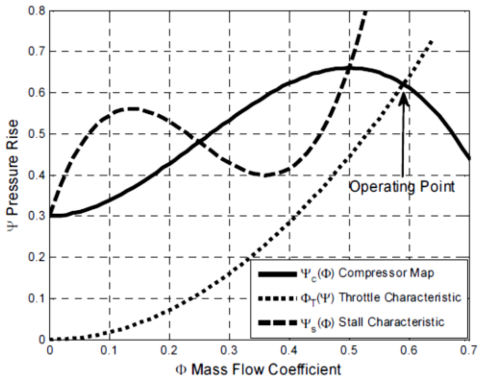

Fig. 3: $\Psi_c(\Phi)$, $\phi_T(\Psi)$ and $\Psi_s(\Phi)$ [4]

III. Model Behavior And Non-Linearities Analysis

A complete bifurcation analysis of the gravdhal model in equation (1,2,3,4) has been carried out [**4**]. The Mancont continuation and bifurcation software [**6**] has been used to carry out the computations. The computed bifurcation diagram, for mass flow Φ with varying throttle parameter γ_T, for the case *Ud=160*, is presented in Figure 4. (Similar bifurcation diagrams for the other variables, J_n and Ψ, can also be plotted.)

Sari et al. [4] developed the bifurcation analysis of Gravdahl's model for non-constant speed axial compressors. Figure 4 shows one of the bifurcation diagrams of the model where equilibria of a non-constant speed axial compression system are depicted as a function of throttle gain (bifurcation parameter). Figure 4 contains information about all steady states and their stability, and identifies bifurcation points where steady states exchange stability, new steady states are created, or existing steady states disappear.

The subcritical bifurcation point represents the peak value of pressure rise where the axisymmetric flow loses stability.

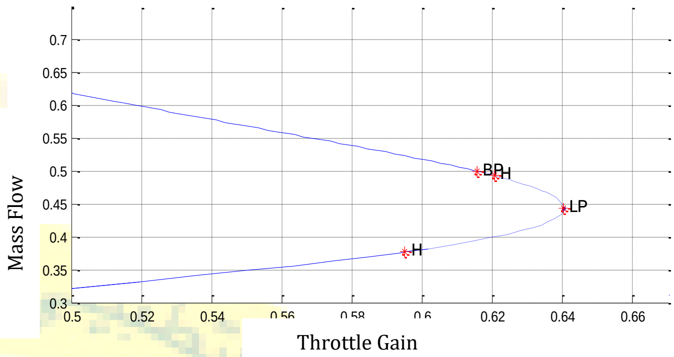

Fig. 4.a: Bifurcation diagram of mass flow

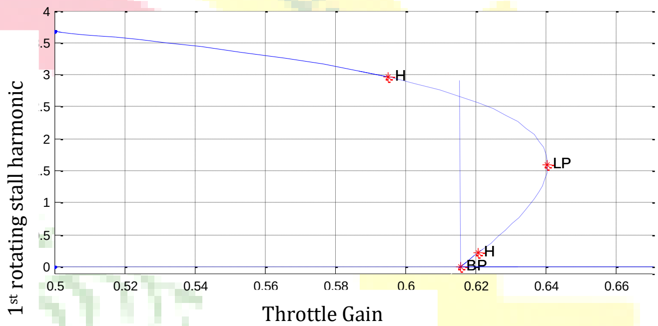

Fig. 4.b: Bifurcation diagram of first stall harmonic

Limit cycles originating from Hopf bifurcation point H (inferior) represent classical surge with *J>0*. Classic surge cycles occur only for a narrow range of values of the throttle parameter γ_T, and are therefore plotted separately in Fig. 4.b on a different parameter axis scale. Limit cycles from Hopf point H (superior) are not plotted because they are found to have negative *R* and are therefore nonphysical. Bifurcation diagrams, such as that in Fig.4, can be used to identify parameter regions of different global stability behavior [7].

IV. Final Dynamic Model Of Variable Speed Axial Compressor

Gravdahl developed a model for variable speed axial compressors and considered the speed of the rotor as a state variable [2]. Later, Zaiet et al. [8] modified the model to include the pressure drop over a CCV and to make it suitable for control applications.

At an operating point (Φ_0,Ψ_0, U_0), the dynamic model can be given in the form of state-space equations in error coordinates (see [4,2] for details). The model which is only includes the first harmonic of rotating stall and comprises actuator forces, is given in the following equations:

$$\frac{d\Phi}{d\xi} = \frac{H}{l_c(U)}\left[-\frac{\Psi-\psi_{c0}}{H} + 1 + \frac{3}{2}\left(\frac{\Phi}{W}-1\right)\left(1-\frac{J_1}{2}\right) - \frac{1}{2}\left(\frac{\Phi}{W}-1\right)^3 - \frac{u_1}{H} - \frac{1}{{\gamma_v}^2}C_1J_1 - G_1\Phi u_3 + G_1K\Phi^3 + \Delta_\Phi\right] \quad (13)$$

$$\frac{dJ_1}{d\xi} = J_1\left[1-\left(\frac{\Phi}{W}-1\right)^2 - \frac{J_1}{4} - G_2 - G_3u_3 + G_3K\Phi^2 - \frac{1}{{\gamma_v}^2}C_2\Phi\right]\frac{3aHn}{(n-m_U(U)a)W} \quad (14)$$

$$\frac{d\Psi}{d\xi} = \frac{\Lambda_2 b}{U}\left(\Phi - u_2\sqrt{\Psi}\right) - \frac{2\Lambda_1 U\Psi}{b}u_1 + \frac{2\Lambda_1 KU\Psi\Phi^2}{b} + \Delta_\Psi \quad (15)$$

$$\frac{dU}{d\xi} = \frac{\Lambda_1 U^2}{b}u_3 - \frac{\Lambda_1 K\Phi^2U^2}{b} \quad (16)$$

With

$$C_1 = \frac{W^2}{2H},\ C_2 = \frac{4W}{3H},\ G_1 = \frac{U_d\Lambda_1\Gamma l_E}{bH},\ G_2 = \frac{\mu W}{3aH},\ G_3 = \frac{2U_d\Lambda_1\Gamma(m-1)W}{3Hb},\ G_4 = \frac{3aH}{W} \quad (17)$$

Where Φ, Ψ and U denote respectively the annulus averaged mass flow coefficient, the non-dimensional plenum pressure, and the speed of the rotor. J_1 is the squared amplitude of the first harmonic of rotating stall. The actuators' forces are input variables u_1, u_2 and u_3 defined respectively as: the pressure drop over CCV, the throttle gain, and the non-dimensional drive torque being used to increase the speed. The definition of the remaining model parameters H, W, ψ_{c0}, γ_v, Λ_1, Λ_2, m, b, μ and a, which are all positive non-zero parameters, can be found in [4]. To investigate the effect of uncertainties, we introduce Δ_Φ and Δ_Ψ in the model. Δ_Φ consists of two terms : Φ_d which is a time varying mass flow disturbance and Φ_d which introduces a constant or slow varying uncertainty in the throttle characteristic. Similarly, Δ_Ψ consists of two terms: Ψ_d which is a time varying pressure disturbance and d_Ψ which can be thought of as a constant or slow varying uncertainty in the compressor map. Furthermore, it is supposed that these uncertain terms are bounded.

V. Control Design

Let us consider the model (13,14,15,16) as a square MIMO nonlinear affine uncertain system:

$$\Sigma 1: \dot{x} = f(x) + \sum_{i=1}^{m} g_i(x)u_i \quad (18)$$

where the state variable $x = (\phi, J, \Psi, U)$ belongs to $\mathbb{R}^4$ and the control input $u = (u_1, u_2, u_3) \in \mathbb{R}^3$ Here, f(x)and g(x) are uncertain smooth functions and $y = (\phi, \Psi, U) \in \mathbb{R}^3$ is a smooth measurable output vector. $\Sigma 1$ is defined in error coordinates and in the regulation problem, the objective is to make the outputs vanish in finite time [9].

A. *Control of Surge and Rotating Stall*

Let us define outputs for system $\Sigma 1$ as follows:

$$y_{1,2} = [y_1, y_2]^T = [\phi, \psi]^T \quad (19)$$

Where $\phi = \Phi - \Phi_0$, $\psi = \Psi - \Psi_0$ with Φ_0 and Ψ_0 is the efficient Operating Point at the peak of the compressor map.

Here, the first time derivative of sliding variables yields:

$$\Sigma 2: [\dot{y}_1(x), \dot{y}_2(x)]^T = A(x) + B(x)u \quad (20)$$

Vector A(x) and matrix B(x) can be partitioned into nominal and unknown parts as follows:

$$\begin{cases} A(x) = \bar{A}(x) + \Delta_A(x) \\ B(x) = \bar{B}(x) + \Delta_B(x) \end{cases} \quad (21)$$

Nominal parts $(\bar{A}(x), \bar{B}(x))$and are known a priori. Δ_A and Δ_B traditionally comprise the model uncertainties and perturbations. In this work, however we consider all terms comprising J in Δ_A. Although J is a model state variable, it cannot be measured, moreover its nature as a perturbation conveys the idea that it can be thought of as uncertain terms. This approach simplifies the control design and makes the proposed control method applicable.

Assumption 1: We assume that there is an a priori known constant ρ such boundedness of Δ_A is ensured.

Assumption 2: We assume that matrix $\bar{B}(x)$ is nonsingular and the associated zero dynamics of $\sum 2$ are asymptotically table.

Assume that Assumptions 1-2 are fulfilled. Then, the control law:

$$u_i = \bar{B}^{-1}\left(-\bar{A} + W_{i,nom} + W_{i,slid}\right) \quad i = 1,2 \quad (22)$$

The terms $W_{i,nom} = -\beta_i y_i$ are introduced to stabilize the nominal part of system ($\Delta_A = 0$), where β_i is a control positive parameter [10].

we define an augmented sliding variable $S_a \in R^2$ and its associated discontinuous control law as follows:

$$S_a(y, S_{aux}) = [y_1, y_2]^T + S_{aux} \quad (23)$$

where auxiliary function $S_{aux} \in R^2$ with $\dot{S}_{aux} = \beta_i y_i$ is used in the design of the augmented sliding variable and discontinuous control law $W_{i,slid}(y, S_{aux}) \in R^2$. The time derivative of along the system trajectories $S_a(y, S_{aux})$can be expressed as:

$$\dot{S}_a = \left[\dot{\phi}, \dot{\psi}\right]^T + \beta_i y_i = W_{i,slid} + \Delta_A \quad (24)$$

The control law stabilizing the nominal system and rejecting the uncertainties of the model takes the final form

$$u_i = \bar{B}^{-1}\left(-\bar{A} - \beta_i y_i - \alpha_i \operatorname{sign}\left(\beta_i y_i + \int \beta_i y_i\right)\right) i = 1,2 \quad (25)$$

α_i is a control positive parameter. For proof, see [11].

B. *Control of variable Speed*

Let us consider the model (16) as a SISO nonlinear affine system.

$$\dot{U} = A_3(\phi^2, U^2) + B_3(U^2)u_3 \quad (26)$$

We designed a sliding surface with good nature and made the system possess the desired properties when make the system limits on the sliding surface. In order to facilitate control, we make the system reach the sliding surface and keep sliding. So, we can define the following tracking error [13].

$$e_3 = U - U_d \quad (27)$$

The speed error can be written as

$$\dot{e}_3 = \dot{U} - \dot{U}_d = A_3(\phi^2, U^2) + B_3(U^2)u_3 - \dot{U}_d \quad (28)$$

Define a time-varying proportional integral sliding mode surface [12].

$$S = \beta_3 e_3 - \int_0^t \beta_3 \, Le_3(\tau)d\tau \qquad (29)$$

With β_i is a control positive parameter, and L is negative value. Under the sliding mode, the equation $S = \dot{S} = 0$ must be satisfied, where

$$\dot{S} = \beta_3 A_3 + \beta_3 B_3 u_3 - \beta_3 \dot{U}_d - \beta_3 Le_3 \qquad (30)$$

To meet the sliding conditions, the following controller is designed

$$u_3 = (-\beta_3 B_3)^{-1}(\beta_3 A_3 - \beta_3 \dot{U}_d - \beta_3 Le_3 + \varepsilon \, sign(S)) \qquad (31)$$

where $sign(S)$ is sign function.
Assume that the constant ε satisfied the condition $\varepsilon > 1$ .Then the system (26) can reach the sliding mode $S = 0$ in a limited time under the controller (31), with a variable speed reference U_d. For proof, see [14].

VI. The Numerical Simulation

In the following time-domain simulations, we simultaneously control the speed and surge/rotating stall which is called closed-loop.
To realize the simulations two types of perturbations are applied to the system denoted by $\phi_d(\xi), \psi_d(\xi)$ are considered as mass flow and pressure disturbances respectively and d_ϕ, d_ψ which represents the uncertainty of compressor map and throttle characteristic.
The numerical values of the simulations are given in Table 1.

Table 1: Numerical values used in simulations

W	0.25	$\phi_d(\xi)=\psi_d(\xi)$	0.01sin (0.2ξ)
H	0.18	l_I	1.75
μ	0.001	d_ϕ	−0.05
b	96.16	d_ψ	0.02
m	1.75	Λ_1	2.1685e-4
l_c	3	Λ_2	0.0189

Figure 5 shows the variables Φ and Ψ in the phase space along with equivalent compressor map and stall characteristic. The system start from an effective initial operating point (OP) at the top of the equivalent compressor map (i.e. compressor comprising CCV).
Examining of the Figure 5, we found that the system dynamic in closed loop, stay close to his efficiency operating point (0.5,0.66) despite the existence of uncertainties, perturbation (negligible variation).

As reported in [4], when speed varies at an efficient operating point (e.g. at the peak of the equivalent compressor map), temporary stall developments can lead to a fully developed rotating stall. Here, we will show that the proposed robust nonlinear controller prevents the system from developing such a rotating stall.

Figure 7 shows that on the other hand in closed loop, the controller effectively stabilizes the compression system at the efficient OP and prevents it from developing a steady rotating stall due to the speed variation.

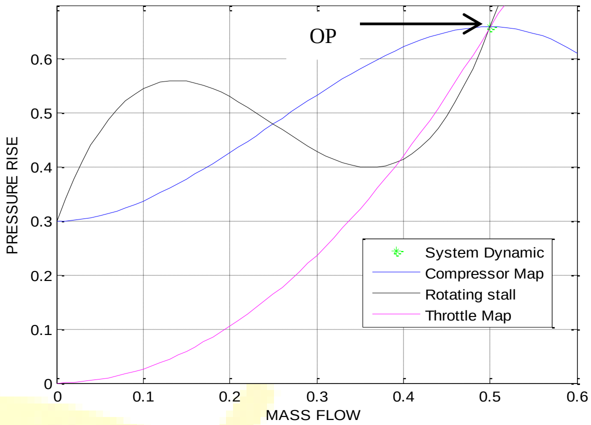

Fig. 5. Closed loop System Map

Previously reported results in [4], show that pressure and flow external perturbations can destroy the stability of compressors at an efficient OP and lead to fully developed rotating stall or deep surge depending on the speed of the rotor (i.e. for low speeds the system goes to rotating stall and for high speeds it develops deep surge). Here, we consider the case of high speed operation (according to [2],[4]) and demonstrate that the controller can effectively reject the perturbations and guarantee the stability of the system.

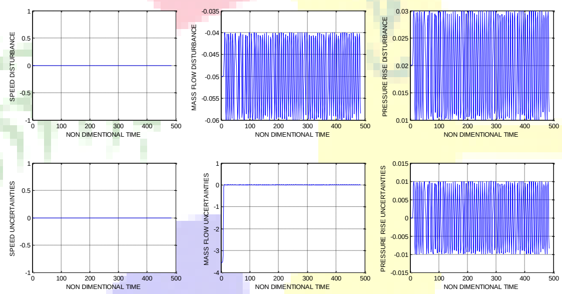

Fig.6.Flow and Pressure uncertainties

Output pressure, rotor speed, rotating stall, and control efforts are respectively reported in figure 7.

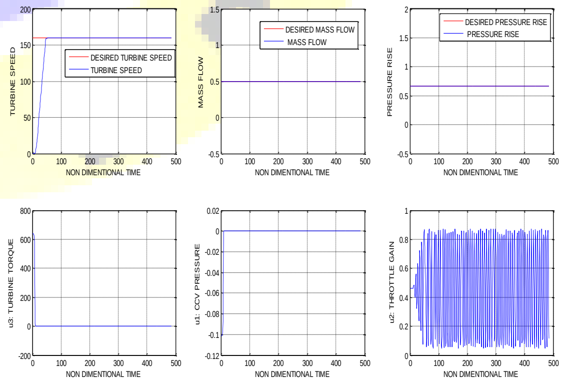

Fig.7. System Dynamic in Closed Loop Robust Sliding Mode Control

At $\xi = 10$, the controller is activated and closes the loop. It immediately damps out rotating stall (Figure 8) and manipulate the throttle valve. Consequently, we note a variation in throttle actuator (Figure 7), despite that the system still close in his efficient OP, where the pressure is high enough for normal operation of the gas turbine.

This Result shows the effectiveness of the proposed control law in surge and rotating stall control.

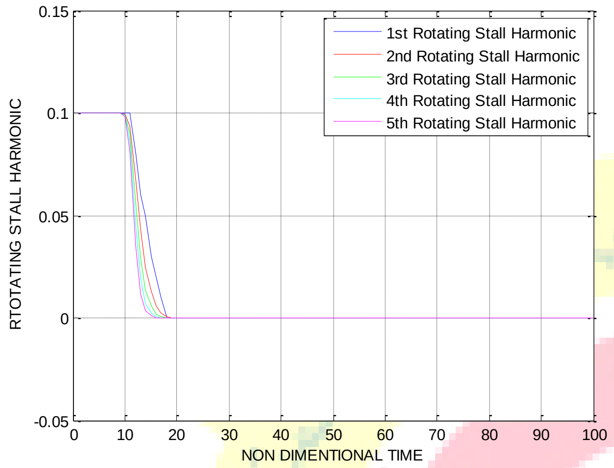

Fig. 8. Rotating Stall Harmonics

The only weakness of this robust nonlinear controller is the existence of variations in throttle valve manipulated control signal, it needs to be carefully investigated.The actuator dynamics and saturations will be treated in future works.

VII. Conclusion

This paper show that a gas turbine, which is variable speed in nature, suffers from temporarily developed instabilities which may lead to a steady and fully developed rotating stall or surge, and reveal the impact of speed transitions (output signal) and throttle gain (control signal) on the stability of compression systems.

By the addition of model uncertainties and external perturbations, and impossibility to have a full feedback control (rotating stall is no measurable) forms a challenging problem.

The robust sliding mode control method based on feedback linearization was the favorite control philosophy to tackle this open control problem. The proposed controller does not require the precise knowledge of the compressor map and does not use a full-state feedback. The only assumption made here is the boundedness of external perturbations and model uncertainties.

Time-domain simulations demonstrate that the controller can damp out system instabilities including surge or rotating stall, prevent the system from developing temporary rotating stall during speed variations and effectively reject external perturbations.

The limitations and the dynamic of the actuators and sensors have not taken into consideration, which needs to be studied further.

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